SESSION 14: Undergraduate Research in Computer Sciences

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14.1: Computer Analysis of Musical Style

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GENERALLY, style is a unique method of organization of artistic material. This paper is the result of some attempts to find, using the computer, an objective measure of style in music which would be generally acceptable, and which would not conflict with previous, subjective measures*. A large number of piano scores have been analyzed by computer for parameters such as melodic autocorrelation, chord structure, chord duration, chord type, key and modulation, with a view to obtaining an insight into the stylistic differences between the works of Haydn, Mozart and Beethoven. It is not assumed that measurements related to these and other parameters are necessarily indicative of style. The purpose of making them is to determine whether and to what extent they correlate with more subjective notions of musical style.

Previous Results on Statistical Analysis of Music

An extensive statistical analysis is reported by Fucks[2], who analyzed a large sample of music over a wide historical range (1600 to the present), with regard to (1) - pitch, and (2) - interval between two simultaneous parts. For (1), duration, and interval between consecutive pitches were considered. Only the first violin part for orchestra music and the soprano part for choral music was analyzed for the pitch statistics. For (2), intervals between the first and second violin parts in orchestra music and between the soprano and alto parts in choral music were analyzed. Statistical parameters such as mean, second, third and fourth moments, skew, and excess (as defined below) were noted. Of these, the last seemed to have certain merit as a style parameter, when applied to the distribution of frequency of occurrence versus interval between consecutive pitches in the sample melodies.

$$\text{Excess} = \frac{\mu_4}{\sigma^4} - 3$$

where μ_4 is the fourth moment and σ is the mean of the distribution.

This parameter measures the steepness or peakedness of the distribution relative to the Gaussian probability distribution for which the excess is always zero. Plotted against historical time, it is reported to show an almost linear growth from the Baroque era to the contemporary, non-serial era, increasing by a factor of four after which there is a sharp drop with contemporary serial composers.

Fucks also plotted graphs of the autocorrelation function of the melody which he called "correlograms". Considering the melody to be a series of n notes x(i), one can compute correlation coefficients between x(i) and x(i+k), i = 1, 2, ..., n-k for each of k = 0, 1, 2, ..., 20. A plot of the correlation coefficients R(n,k) versus k = 0, 1, 2, ..., 20 would yield a correlogram. R(n,k) is defined in figure 1, using this notation.

According to Fucks, the correlogram for the sample of Beethoven studied falls slowly with k, and is still positive at k = 20. The one for Bach falls more rapidly and shows some oscillation between small positive and negative values. The curve for Webern drops abruptly to zero at about k = 2 and shows more oscillation than Bach's.

Robert Baker[1] made a statistical study of the chords found in samples drawn from the Haydn, Mozart and Beethoven string quartets. The samples chosen were all in major mode, common metre and were non-modulatory. Three main variables were considered: (1) - harmony type; (2) - note duration; and (3) - chord position within the measure (i.e., 1st, 2nd, 3rd or 4th beat).

Dependent relations between variables were investigated, using chi-square tests of significance. One result which confirms previous observation is that Haydn tends to change his harmony more frequently than either of the other two composers.

Preliminary Experiments

Previous computer analysis of music required that certain parameters, such as key and harmony, be manually extracted from the music. Since this method requires traditional harmonic symbols, it is therefore restricted in its application. What is needed is a coding method which does not require a traditional pre-coding. For my analysis, I coded directly the actual notes and durations in the musical scores themselves.

Input Scheme

The music coding system I used was intended to be a means of obtaining the pitch and duration information of a musical score in digital form suitable for computer processing. This coding scheme specifies each note of the musical composition as to pitch and duration, but subjective quantities, such as indications of key or instructions to the performer, are omitted. It is analogous to the traditional musical notation, and to master it requires only an ability to read music. It was possible to code the melody line separately so that it could be easily retrieved for analysis. This was not done in later work since it was not needed for an analysis of harmony and would unduly slow down the coding. The system can be adapted to music for any instrument or combination of instruments, but it has been used primarily for coding piano scores. There is a one-to-one correspondence between measures and "bar-cards", and the music can be keypunched directly from the scores.

As an example of its large-scale use, one assistant** working for a month and a half was able to encode virtually all the pianoforte sonatas of Haydn and Mozart in common meter which were practicable under the present coding system, plus a comparable number of measures from Beethoven sonatas, which are longer and more numerous. These data, which have been checked for consistency and keypunching errors by an editing program, represent a total of nearly 5,000 bars of music, or over 15,500 "bar-cards". However, it must be appreciated that the task is not simply a matter of keypunching, as it involves a translation from the musical notation to an alpha-numerical code. But the example is an adequate demonstration that large quantities of such coding could be performed readily if needed, thus removing one possible bottleneck in the effective use of the computer as a music analyzing tool.

Preliminary statistical tests were performed on some Brahms intermezzos and Bartok piano pieces. These tests fall into three main categories, and were performed separately for each section (or movement) of data studied.

(1) - Note Counts and Interval Structure

These measurements involve counts of the pitches and pitch combinations which were found to reflect certain aspects of the composer's style.

(a) - The distribution of pitches was calculated using a twelve-tone basis and ignoring octave duplication. It was found that the computer could determine automatically the key in which the sample was written by comparing this distribution with arbitrarily assumed distributions for all twenty-four possible keys. By this method it was possible to determine the key with perfect accuracy over the range of musical examples tested. Once the key had been discovered, the pitch distribution could be shifted cyclically into a standard key, either C major or A minor for direct comparison with other samples.

(b) - A distribution of the notes used also was tabulated using an eighty-eight note basis, along with its mean, scatter, and second, third and fourth moments.

(c) - A table was assembled in which each different combination of intervals was printed once along with the percentage of the total time it was used. Each chord structure was tabulated as a group of integers arranged vertically, where each integer denoted an interval by number of semitones above a common bass note. Thus a major triad in root position and close structure would be coded as $\genfrac{}{}{0pt}{}{7}{4}$. Following the terminology of Schillinger[3], these interval structures will be referred to as $\sum$'s.

(d) - From this table a distribution of the intervals (above the bass note) was tabulated, together with the mean and standard deviation. Typical values would be 15-20 semitones for the mean and 8-11 semitones for the standard deviation. It was expected that this measure of interval range would be useful as a historical style parameter, and would be smaller, for example, for harpsichord music than for piano music due to the greater range of the latter instrument. This expectation was confirmed in later experiments.

All the above measurements were performed in two ways: weighted by duration, and weighted by frequency of occurrence.

Little difference was noted between the results corresponding to the two methods of weighting. This result was used in future analyses, which were all weighted by duration.

(2) - Melodic Content: Skips and Correlograms

(a) - Distributions were tabulated of the consecutive skips (interval between successive notes) occurring in the melody line. Some melodies showed a preference for odd-semitone skips but others showed a large percentage of repeated notes and downward whole-tone skips; Figure 2.

A typical value of σ for the skip distribution would be about 4-6 semitones. The mean is made exactly zero instead of approximately so by considering the last note of the melody as proceeding circularly to the first.

The excesses of these distributions turned out to be quite high, in confirmation of the previous work mentioned. Values of 3 to 7 were common, except in a few cases in which the melody is composed chiefly of large skips***.

(b) - Following Fucks, correlograms were plotted for the sample melodies. For Brahms these were relatively smooth and well-behaved, but for Bartok they had a more scattered appearance, a result most likely due to the relatively short length of the samples involved. The Brahms opus 116 number 5 correlogram dropped to zero at about $k = 10$ and stayed there showing only small oscillations. However, it was more common for the correlograms to oscillate, for example the one for the Brahms intermezzo opus 116 number 3 section 3. This one shows a large positive correlation at $k = 0$, 8, and 16 and a large negative correlation at $k = 4$, 12 and 20. The reason for this form of oscillation is strongly suggested by the arpeggios in the piano score.

(3) - Root Cycles

The root of a chord is the note on which the chord is built, and may be labelled I, II, ..., VII in the diatonic scale. This is intuitively a simple concept, but it is difficult to give an operational definition that will work in most cases. The method which I developed for extracting the root of a chord is based on the diatonic scale, and is independent of accidentals or chromatic tones. Thus the method is dependent on the notation of the composer, who is usually careful in his delineation of harmony, and is more generally applicable to tonal music than would be a procedure employing a twelve-tone basis.

We place the seven tones of the scale in a circle of thirds as in Figure 3. Then by proceeding clockwise around the circumference, we can construct a chord from any starting point as a root. For example, consider the chord CEG as illustrated in Figure 3. This triad may be major, minor, diminished or augmented, but has the same root, C, in all cases.

Thus the root may be defined as the note furthest counterclockwise around the circumference in the sequence of labelled notes. There is, however, some difficulty in applying this definition, since it is not always clear which is the note "furthest counterclockwise". In practice, two mutually exclusive situations can occur which must be accounted for separately; Figure 4.

We might, for instance, seek the largest unmarked arc of the circumference, calling its clockwise extremity the root. For instance in the first case illustrated in Figure 4, the largest open arc is between D and C, therefore C would be the root. But in the second case, there is no largest open arc, and the definition of root breaks down. Rather than admit there is no root, we can look for another defining principle. Here I chose D as the most likely root, since it appears with a third above. In this type of situation we look for the widest marked arc of the circle, and call its counterclockwise extremity the root. Thus the alternate criterion is a kind of inversion of the first.

Of course, in the unlikely event that all seven tones appear simultaneously, there will be no root.

Distributions showing the percentage use of the six possible diatonic root cycles were computed for each section. In the notation of Schillinger[3], cycle 3 or C_3 occurs when the root moves down a third (or up a sixth), and similarly for C_5, C_7, C_{-3} and C_{-7}. These define all possible root movements, and their use was shown by Schillinger to be a significant style parameter. Root cycles were tabulated bar by bar for each section, and a summary was made of the percentage use of each for the whole section. In this tabulation, some passing chords which would normally not be analyzed for a separate root are given equal weight with the predominant chords of longer duration. If desired, these could be screened out by a separate test but this would be to negate the advantage of the computer in this application, namely its ability to sort out the minute details which an ordinary analyst would tend to ignore.

The root cycle distributions for Brahms were relatively consistent over the range of samples analyzed but were in sharp contrast with those for Bach, in explicit verification of previous notions. This test seems to show much promise as an indicator of style, and should be investigated in more detail.

Later Experiments

Following Baker, a new project was initiated to investigate and **compare** the harmonies found in the piano sonatas of Haydn, Mozart and Beethoven. Piano sonatas were chosen rather than the string quartets since the former were readily available and the latter had already been **analyzed** statistically by Baker[7].

Algorithms were devised which can detect key and key change (modulation) within a section or movement, which can find the inversion of a chord (position of the root) using only its twelve-tone representation, and which can classify chords as major, minor, or diminished or dominant seventh. These were coded to show that a machine could perform automatically the same kind of analysis previously done by hand, and to make the results more readable by a musician. Since all the information about the chord is contained in its interval structure, the classification where possible into traditional symbols was done only to prove a point and not to increase the value of the information. Strong and weak beats were indicated as functions of bar position, and the results were tabulated in a form which allows a direct comparison of the harmonic structures used by the three composers chosen.

In piano music of the style being considered, a great deal of stylistic information may be contained in the ornamentation of melody. The ornaments, which include turns, mordents, trills, and so on, are superimposed on the melody line, and were originally used to prolong the force of the sustained notes, particularly in early piano and harpsichord music, since the sound of a harpsichord dies away rapidly after the key has been struck, compared with the tone of a modern piano. Variations in the way these ornaments are used by different composers could assist a trained listener in recognizing the composer's style. The variations might include: the actual form of the figuring, and the voice in which is occurs. In the present harmonic analysis, ornamentation as such was ignored, although it was coded into the appropriate voice wherever it occurred. The analysis of melody and ornamentation should be performed as a separate study in itself, although it was not attempted here.

The analysis performed falls into three divisions: interval structure and harmony, key and modulation, and root and inversion. The results of the analysis were accumulated and sorted in such a way that for the entire sample of data they could be compared directly between the three composers.

Interval Structure and Harmony

The chief purpose of this experiment was to provide data on the use of interval structure and harmony in Haydn, Mozart and Beethoven. Having numerically specified the pitch and duration of each note, combinations of simultaneous notes could be considered at any point in the score. Such a group or $\sum$ may conveniently be represented as a series of intervals above the bass, or lowest note. In this way, the explicit structure is retained, not simply categorized. It is here that the important stylistic information resides, not in the mere chord type. Style is conveyed by the specific variations which occur within the traditional harmonic classification.

It is desirable to be able to compare directly the harmonies used by Haydn, Mozart and Beethoven to discover which were used most frequently by each composer, and more important, to pick out the important differences which might provide an insight into their stylistic individuality. For example, it is generally expected that Beethoven would use intervals spanning a wider range than either Haydn or Mozart since he wrote for a larger piano.

Results of this comparison show that the octave (12) is by far the most common $\sum$, and is used nearly twice as much by Haydn as by either Mozart **or** Beethoven. The minor and major thirds (3 and 4, respectively) were used next most frequently, again more by Haydn than by Mozart or Beethoven. Particular three-note chords were less common, since there is more possibility of variety. The minor chord $^{15}_{3}$ for example, was used about one tenth as much as the major third (4), and more often by Beethoven than by Haydn or Mozart. In general, it was observed that Beethoven used more notes per chord than did either of the other two composers.

One might expect that a limited number of different chord structures would appear in the works of each composer. That is, a graph of the number of different chords found plotted against the chord number analyzed might be expected to rise sharply at first, with a slope approaching unity, and then somehow decrease in slope and eventually level off. In the present analysis, however, chord structure was complicated by chord function, so that a single chord form might be found under several classifications depending on the degree of the **scale used, and whether** it occurred on a strong or weak beat. Thus the number of possibilities for different chords was greatly increased over the number obtained only from differences in interval structure and discounting secondary classifications resulting from key and position within the measure. The amount of this increase has been measured, using the data available, and was found to be about 220%. Due partly to this increase in number of separately distinguished chords, the graph of number of different chords versus chord number considered shows no sign of levelling off after the initial rapid increase, but appears to continue rising with constant slope in all three cases corresponding to the three composers analyzed; Figure 5.

The curve for Beethoven is distinctly higher than those for Haydn and Mozart, rising with a steeper slope. A related measurement is the number of different harmonies per harmony used. This number is simply the slope of the line joining the origin to a point on the graph, and in general decreases with increasing sample size. Values are tabulated below for the total available samples.

Haydn	.22
Mozart	.29
Beethoven	.40

Values of the slopes of the straighter portions of the curves would have indicated the same general trend, which is a reflection of the fact that Beethoven's harmonic texture is thicker than Haydn's or Mozart's. Thus a greater harmonic variety is to be found in a sample of Beethoven, compared with samples of Haydn or Mozart of similar length.

Another observation of interest is the average number of time units per harmonic unit. This measurement confirms the observation, previously verified by Baker, that Haydn tends to change his harmonies more frequently than either Mozart or Beethoven. The two sets of results are presented here, for comparison.

Baker

Haydn	3.95	eighth-notes per harmonic unit
Mozart. . . .	4.25	eighth-notes per harmonic unit
Beethoven . .	4.21	eighth-notes per harmonic unit

Gabura

Haydn	1.67	sixteenth-notes per harmonic unit
Mozart. . . .	1.73	sixteenth-notes per harmonic unit
Beethoven . .	1.73	sixteenth-notes per harmonic unit

The discrepancy in the size of the numbers is in part a result of the fact that in the present analysis, smaller harmonic units are acknowledged as being distinct. The important thing to observe is that the figures are smaller for Haydn than for Mozart or Beethoven.

<u>Key and Modulation</u>

The automatic sensing of modulation is required so that chord function may be compared in different contexts. The same $\sum$, for example, might be used with different roots in the same key, or might serve simultaneous functions in different keys, for example during a modulation. I was encouraged here by the positive success scored previously with respect to the sensing of key. The sensing of key change within a movement was therefore thought feasible, but it was realized that this would be a more delicate matter, particularly since it is a question of intrinsic and deliberate ambiguity. Some sort of weighting was seen to be necessary which would, for any point in the score, weight the preceding chords with decreasing importance. Chords were used as the positional weighting markers rather than measures or beats, since this would insure that enough notes were present in the count to even out the fluctuations which might result from intervening rests or solo passages. Arbitrarily, a decreasing exponential weighting into the past was used; Figure 6. Initially ten chords were considered together, but it was decided to use twenty in an attempt to even out the key change fluctuations. The weighting factors decreased by a total factor of ten from the chord being considered to the twentieth previous chord. In addition to the exponential weighting the chords were also weighted by their durations. The notes in these chords were weighted and the resulting twelve-note distribution compared with arbitrarily assumed distributions for all twenty-four keys. It was discovered that by replacing the arbitrary test distributions with the definitions of key from elementary music theory, with ones in the diatonic and zeros in the chromatic scale positions, spurious key changes could be avoided.

In the course of a modulation, it is the intention of the composer to produce a certain ambiguity of key. However it was observed that with the parameters adequately adjusted, the computer indicated key changes decisively without oscillation between the two keys present. This experiment was highly successful, and can be extended to account for the ambiguity of key which occurs in modulatory passages by indicating two or three predominant keys and the "percentage" of each present according to the weighting procedure used. This added flexibility acknowledges a multiplicity of chord function, and might provide information about the possible stylistic differences in the way modulations are effected by different composers.

The method might also be applied to "atonal" music in an attempt to determine how much tonality, if any, is actually present. An orchestral work by Stravinsky is being coded and analyzed in an effort to improve on previous attempts at analysis which were both clumsy and arbitrary.

Root and Inversion

The concept of root is best understood in relation to the diatonic scale and is difficult to define even in the seven-tone basis. In this case it was necessary to determine the root from a group of integers representing notes in the twelve-tone basis. A method was found which works well enough to be applicable to the style of music being analyzed. The system will find the roots of major and minor triads which may be extended by the addition of thirds in an alternating major-minor sequence. That is, the chord may possess an added seventh, ninth, and so on as long as the complete chord can be represented as a sequence of alternate major and minor thirds. A twenty-four element tone wheel was found to be useful as the basis of an operational definition of root in the twelve-tone system.

Consider the circle of alternating major and minor thirds in Figure 7. Each of the twelve chromatic tones appears twice, once on the inner and once on the outer circumference. Given an arbitrary selection of notes, we want to find the root if one exists. The root of course must be contained in the chord, although the additional tones need not be. By the present definition, a chord is built up by beginning at some point on the circumference and proceeding clockwise around the circle, adding additional tones at will. Thus reversing the process, we look for the point at which this series begins, calling the note at the extreme counterclockwise position the fundamental root. This may be accomplished by the following procedure:

(1) - Mark all the different notes appearing in the chord. The notes will appear in pairs, one on the inner and one on the outer circumference.

(2) - With each note thus marked, associate a number which will be the number of positions along the circumference to its nearest marked neighbor.

(3) - Now consider each pair of notes, and delete the mark from the member of the pair which has the greatest number associated with it.

(4) - Compute new "proximity indices" for all members of pairs yet intact, and repeat step 3. If one member of each pair has been deleted, then proceed, otherwise repeat steps 3 and 4 a sufficient number of times (5 or 6) before admitting that no root exists by this definition.

(5) - Compute the sizes of the arcs of the circumference between all adjacent pairs of notes. The note at the clockwise extremity of the widest arc is then the root.

Having extracted the root, the chord inversion may be determined by noting the position of the bass note or lowest sounding note with respect to the root, along the circumference of the "tone wheel".

This method will not produce roots for certain chords, but these may be found by additional tests. Of course a foolproof method of extracting roots may be constructed using separate tests, but it is more interesting to try to work from basic principles.

The root analysis, like the analysis of chord structures, is valid in all contexts, and could be used as the basis of a simpler composer model than that obtainable from the chord structure analysis. Its disadvantage is that it relates basically to the diatonic system, and does not apply to atonal music. It could be used to detect diatonic harmony forms in atonal music, and perhaps might even be used as a sort of measure of the percentage tonality of a composition (if by tonality we mean the use of traditional chord forms).

Presentation of Results

The harmony analysis for any individual section of data could be printed out if desired. This output would list the chord forms in the sequence in which they occurred, and their durations. Information was also provided concerning bar number, strong or weak beat according to bar position, the key to which each chord was most closely related in the context, the root, inversion and chord type, the position of the bass with respect to the key. The information was arranged in vertical columns in the format illustrated in Figure 8.

These chord parameters may require some further comment.

Beat Strength: This was indicated, in common metre, by a 1 for the beginning of the first and third quarters, and by a 2 for the second and fourth quarters. The remaining twelve bar positions were left unlabelled.

Bar Number: A new bar was assumed to begin every sixteen beats. Thus this number corresponds to the real bar number in common metre where the sixteenth beat is the basic time unit.

Interval Structure: These are given as intervals, in semitones, above a common bass note.

Key: This is the predominant key of the immediate and preceding context of the chord. By following this label by rows (versus time) the modulations may be observed and their measure numbers noted.

Type: This can be major, minor, diminished or dominant seventh, and is produced as a by-product of the root-selection routines.

Root: This is the traditional harmonic notation for the diatonic root, relative to the local predominant key.

Bass: This number specifies the semitone number of the bass or lowest note with respect to the key-tone, which is number one.

Inversion: The chord may be in root position, or in first, second or third inversion. The inversion is also found by the root-selection routines.

Number of Beats: This may be the number of consecutive time-units in the sectional analysis, or the total number of time units in the entire sample representing the composer in question.

Composer: The composer will be Haydn, Mozart or Beethoven.

Conclusion

The techniques of applying computers to the analysis of musical style are still in their exploratory stages. Much work remains to be done before style can be evaluated accurately and objectively by computer. What I have done is to investigate certain objectively definable parameters of music, some of which show promise as indicators of style. I have also developed a music coding system and shown that a traditional analysis of music of this period can be carried out using a computer provided with data consisting of only the pitch and duration information coded directly from the score.

Acknowledgement

I wish to acknowledge gratefully the invaluable support and guidance of C. C. Gotlieb, Director of the Department of Computer Science and of the late M. S. Schaeffer, former Head of the Electronic Music Studio in the Faculty of Music, without which any of my original work would have been impossible.

[1] Baker, R. A., "A Statistical Analysis of the Harmonic Practice of the 18th and Early 19th Centuries," unpublished doctoral dissertation, *Univ. of Illinois;* 1963.

[2] Fucks, W., "Musical Analysis by Mathematics," "Random Sequences," "Music and Accident," *Gravesaner Blatter,* 23/25: pj. 146; 1962.

[3] Schillinger, J., "The Schillinger System of Musical Composition," *Carl Fischer, Inc.;* 1941.

* In this paper, style will usually refer to individual style within the same creative epoch or musical period.

** I am indebted to F. E. Braunlich for his efficient coding assistance.

* Brahms' intermezzo opus 116 number 5 where the melody has an excess of only —.56, indicating a distribution very close to Gaussian. The middle section of the intermezzo opus 116 number 2 has an even more negative excess, —1.2. In fact, this distribution has peaks at plus and minus twelve semi-tones which together account for more than 52% of the total skips.

$$R(n,k) = \frac{\dfrac{\sum x(i)x(i+k)}{(n-k)} - \dfrac{\sum x(i)\sum x(i+k)}{(n-k)^2}}{\sqrt{\left[\dfrac{\sum x(i)^2}{n-k} - \left(\dfrac{\sum x(i)}{n-k}\right)^2\right]\left[\dfrac{\sum x(i+k)^2}{n-k} - \left(\dfrac{\sum x(i+k)}{n-k}\right)^2\right]}}$$

FIGURE 1—**Definition of correlation coefficient R(n, k).**

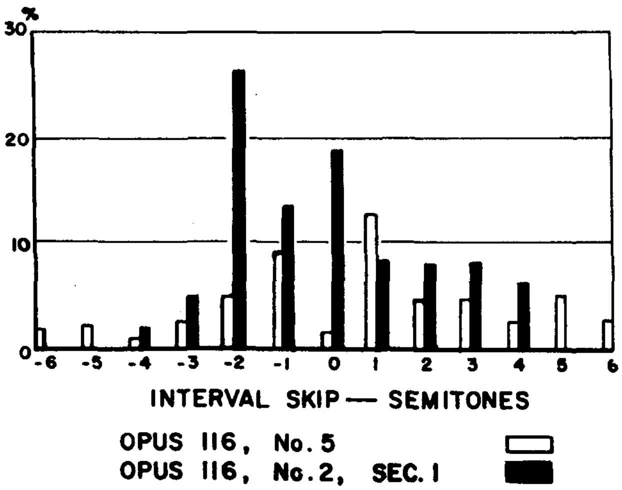

FIGURE 2—**Distribution of consecutive intervals for two Brahms' melodies.**

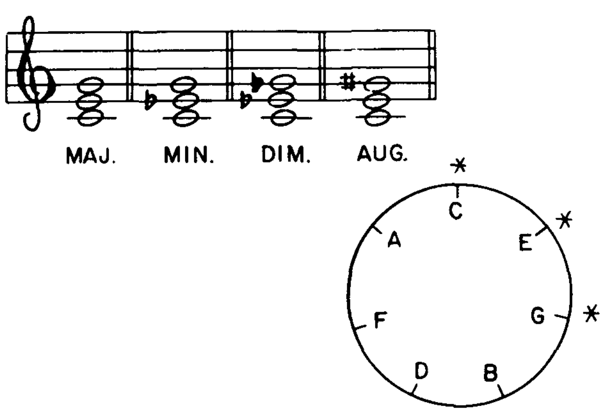

FIGURE 3—**Cycle of thirds for determining the diatonic root of a chord.**

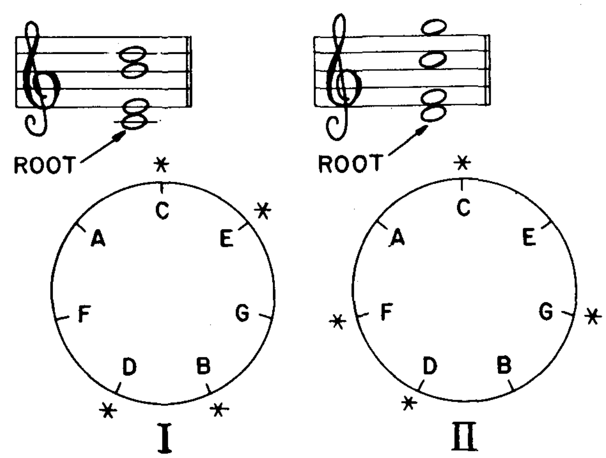

FIGURE 4—**Example of two types of diatonic root.**

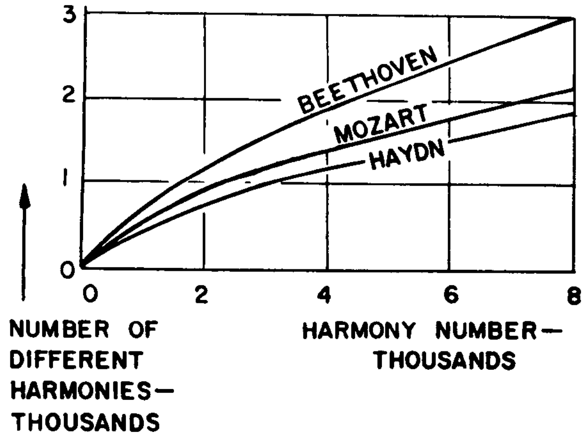

FIGURE 5—Graph of number of different harmonies versus harmony number in the piano sonatas of Haydn, Mozart and Beethoven.

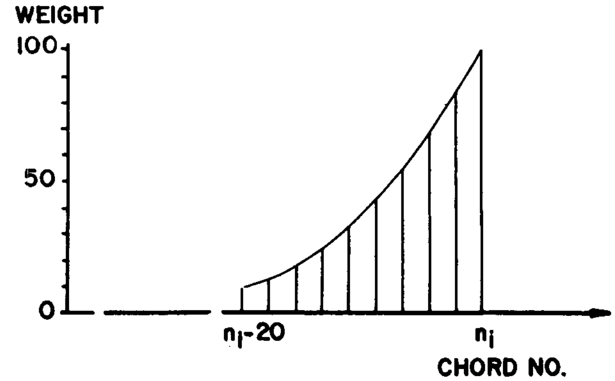

FIGURE 6—Schematic representation of chord weighting for determining key.

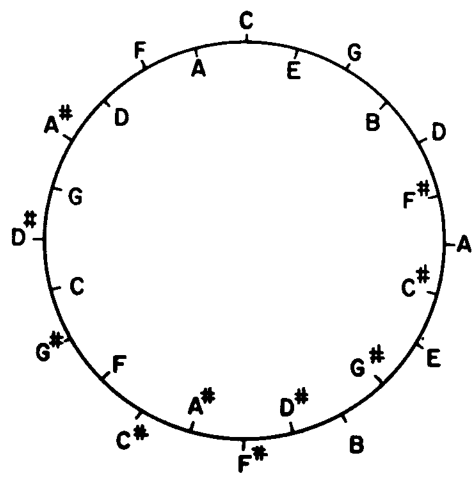

FIGURE 7—Twenty-four element tone wheel used as the basis of a twelve-tone definition of root.

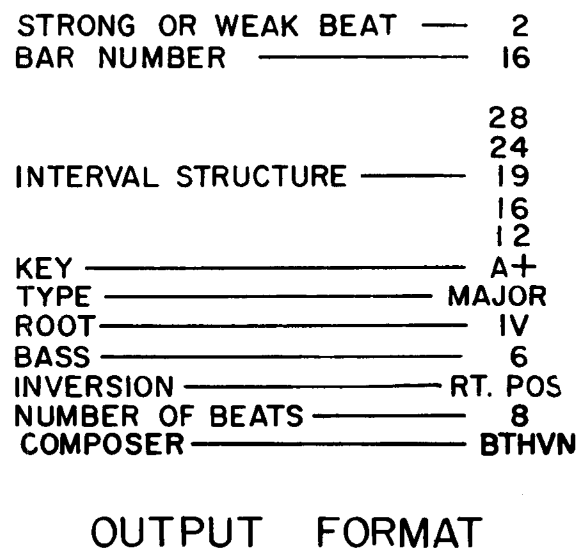

FIGURE 8—Illustration of the output format for the computer analysis of harmony.